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Topic: Applications of Gauss's Theorem

Author: Dr. Amit Kumar Singh

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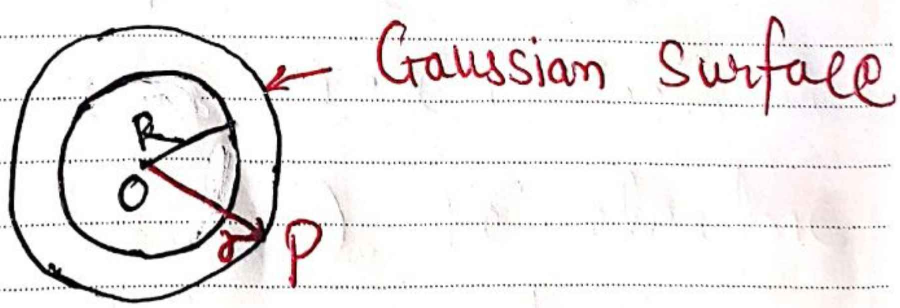
Applications of Gauss's law

It is possible to obtain the electric field in a simple way using the Gauss's law. This can be understood by some examples:

(1) Field outside a uniformly charged Solid sphere of radius R and total charge q:

Let us draw a spherical surface at radius $r > R$; this (fig. 1) is called a "Gaussian surface".



(fig. 1)

Gauss's law says that for this surface

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{q}{\varepsilon_0} \quad (1)$$

Symmetry allows us to extract $\vec{E}$ from under the integral sign: $\vec{E}$ points radially outward, as does $d\vec{a}$, so we can avoid the dot product,

$$\int_S \vec{E} \cdot d\vec{a} = \int_S |\vec{E}| \, da \qquad \text{——} \quad (2)$$

and the magnitude of $\vec{E}$ is constant over the Gaussian surface, so it comes outside the integral

$$\int_S |\vec{E}| \, da = |\vec{E}| \int_S da$$

($\because$ Area of sphere $= 4\pi r^2$)

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$$= |\vec{E}| \cdot 4\pi r^2 \qquad \text{——} \quad (3)$$

Thus, using eqn (1) and (3)

$$|\vec{E}| \, 4\pi r^2 = \frac{q}{\epsilon_0}$$

or,

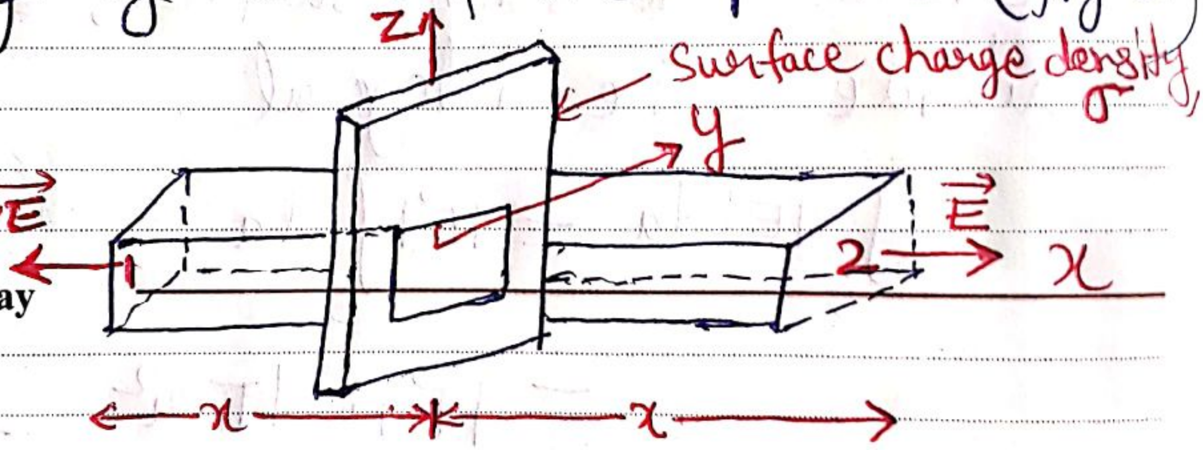
$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot \hat{r}}$$

This means that the field outside the

sphere is exactly the same as it would have been if all the charge had been concentrated at the center.

(2) Field due to a uniformly charged infinite plane sheet:

Let σ be the uniform surface charge density of an infinite plane. (fig. 2)



(fig. 2) → Gaussian surface

By symmetry (from fig. 2), the electric field will not depend on y and z co-ordinates and its direction at every point must be parallel to the x-direction.

We can take the Gaussian surface to be a rectangular parallelopiped of cross section area A.

The unit vector normal to surface 1 is in $-x$ direction while the unit vector normal to surface 2 is in the $+x$ direction. Therefore, flux $\vec{E} \cdot \Delta\vec{S}$ through both the surfaces are equal and add up. Therefore, the net flux through the Gaussian surface is $2|\vec{E}|A$.

or,
$$\int \vec{E} \cdot d\vec{a} = 2A|\vec{E}| \quad \text{--- (1)}$$

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The charge enclosed by the closed surface is σA.

Therefore by Gauss's law –

$$\int \vec{E} \cdot d\vec{a} = \frac{\sigma \cdot A}{\epsilon_0} \quad \text{--- (2)}$$

using eqⁿ (1) and (2),

$$2A|\vec{E}| = \frac{\sigma \cdot A}{\epsilon_0}$$

or,
$$\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0} \cdot \hat{n}}$$

where, $\hat{n}$ is a unit vector pointing away from the surface. $\vec{E}$ is directed away from the plate if σ is positive and toward the plate if σ is negative.